

# INSTRUCTOR'S SOLUTIONS MANUAL

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## THOMAS' CALCULUS EARLY TRANSCENDENTALS FOURTEENTH EDITION

*Based on the original work by*

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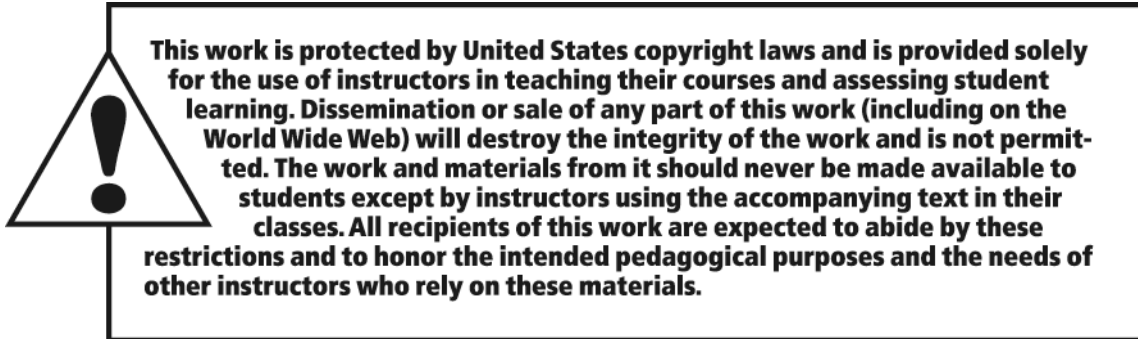
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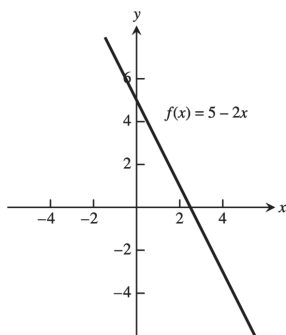
# CHAPTER 1 FUNCTIONS

## 1.1 FUNCTIONS AND THEIR GRAPHS

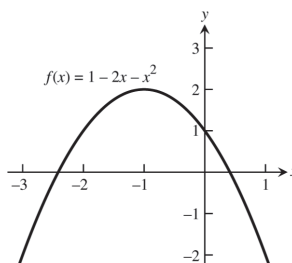
1. domain =  $(-\infty, \infty)$ ; range =  $[1, \infty)$
2. domain =  $[0, \infty)$ ; range =  $(-\infty, 1]$
3. domain =  $[-2, \infty)$ ;  $y$  in range and  $y = \sqrt{5x+10} \geq 0 \Rightarrow y$  can be any positive real number  $\Rightarrow$  range =  $[0, \infty)$ .
4. domain =  $(-\infty, 0] \cup [3, \infty)$ ;  $y$  in range and  $y = \sqrt{x^2 - 3x} \geq 0 \Rightarrow y$  can be any positive real number  $\Rightarrow$  range =  $[0, \infty)$ .
5. domain =  $(-\infty, 3) \cup (3, \infty)$ ;  $y$  in range and  $y = \frac{4}{3-t}$ , now if  $t < 3 \Rightarrow 3-t > 0 \Rightarrow \frac{4}{3-t} > 0$ , or if  $t > 3 \Rightarrow 3-t < 0 \Rightarrow \frac{4}{3-t} < 0 \Rightarrow y$  can be any nonzero real number  $\Rightarrow$  range =  $(-\infty, 0) \cup (0, \infty)$ .
6. domain =  $(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$ ;  $y$  in range and  $y = \frac{2}{t^2-16}$ , now if  $t < -4 \Rightarrow t^2 - 16 > 0 \Rightarrow \frac{2}{t^2-16} > 0$ , or if  $-4 < t < 4 \Rightarrow -16 \leq t^2 - 16 < 0 \Rightarrow -\frac{2}{16} \geq \frac{2}{t^2-16}$ , or if  $t > 4 \Rightarrow t^2 - 16 > 0 \Rightarrow \frac{2}{t^2-16} > 0 \Rightarrow y$  can be any nonzero real number  $\Rightarrow$  range =  $(-\infty, -\frac{1}{8}] \cup (0, \infty)$ .
7. (a) Not the graph of a function of  $x$  since it fails the vertical line test.  
(b) Is the graph of a function of  $x$  since any vertical line intersects the graph at most once.
8. (a) Not the graph of a function of  $x$  since it fails the vertical line test.  
(b) Not the graph of a function of  $x$  since it fails the vertical line test.
9. base =  $x$ ;  $(\text{height})^2 + \left(\frac{x}{2}\right)^2 = x^2 \Rightarrow \text{height} = \frac{\sqrt{3}}{2}x$ ; area is  $a(x) = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(x)\left(\frac{\sqrt{3}}{2}x\right) = \frac{\sqrt{3}}{4}x^2$ ;  
perimeter is  $p(x) = x + x + x = 3x$ .
10.  $s = \text{side length} \Rightarrow s^2 + s^2 = d^2 \Rightarrow s = \frac{d}{\sqrt{2}}$ ; and area is  $a = s^2 \Rightarrow a = \frac{1}{2}d^2$
11. Let  $D = \text{diagonal length of a face of the cube}$  and  $\ell = \text{the length of an edge}$ . Then  $\ell^2 + D^2 = d^2$  and  $D^2 = 2\ell^2 \Rightarrow 3\ell^2 = d^2 \Rightarrow \ell = \frac{d}{\sqrt{3}}$ . The surface area is  $6\ell^2 = \frac{6d^2}{3} = 2d^2$  and the volume is  $\ell^3 = \left(\frac{d}{\sqrt{3}}\right)^3 = \frac{d^3}{3\sqrt{3}}$ .
12. The coordinates of  $P$  are  $(x, \sqrt{x})$  so the slope of the line joining  $P$  to the origin is  $m = \frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}} (x > 0)$ .  
Thus,  $(x, \sqrt{x}) = \left(\frac{1}{m^2}, \frac{1}{m}\right)$ .
13.  $2x + 4y = 5 \Rightarrow y = -\frac{1}{2}x + \frac{5}{4}$ ;  $L = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + \left(-\frac{1}{2}x + \frac{5}{4}\right)^2} = \sqrt{x^2 + \frac{1}{4}x^2 - \frac{5}{4}x + \frac{25}{16}}$   
 $= \sqrt{\frac{5}{4}x^2 - \frac{5}{4}x + \frac{25}{16}} = \sqrt{\frac{20x^2 - 20x + 25}{16}} = \frac{\sqrt{20x^2 - 20x + 25}}{4}$
14.  $y = \sqrt{x-3} \Rightarrow y^2 + 3 = x$ ;  $L = \sqrt{(x-4)^2 + (y-0)^2} = \sqrt{(y^2 + 3 - 4)^2 + y^2} = \sqrt{(y^2 - 1)^2 + y^2}$   
 $= \sqrt{y^4 - 2y^2 + 1 + y^2} = \sqrt{y^4 - y^2 + 1}$

2 Chapter 1 Functions

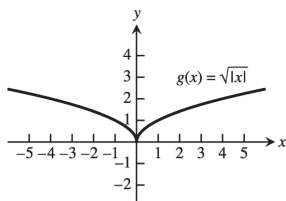
15. The domain is  $(-\infty, \infty)$ .



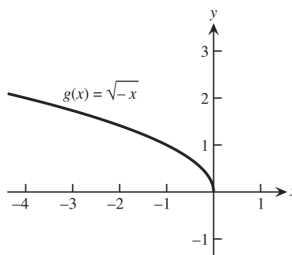
16. The domain is  $(-\infty, \infty)$ .



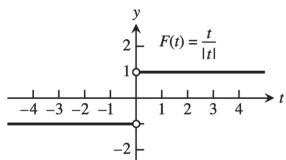
17. The domain is  $(-\infty, \infty)$ .



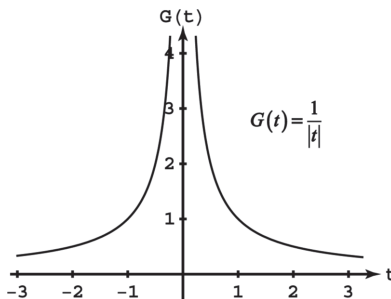
18. The domain is  $(-\infty, 0]$ .



19. The domain is  $(-\infty, 0) \cup (0, \infty)$ .



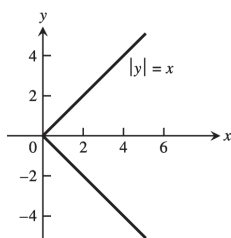
20. The domain is  $(-\infty, 0) \cup (0, \infty)$ .



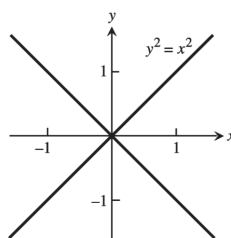
21. The domain is  $(-\infty, -5) \cup (-5, -3] \cup [3, 5) \cup (5, \infty)$  22. The range is  $[5, \infty)$ .

23. Neither graph passes the vertical line test

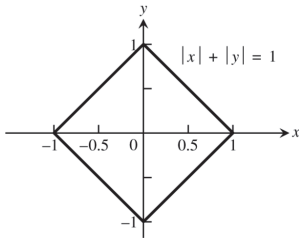
(a)



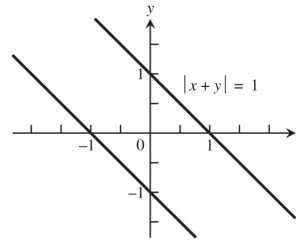
(b)



24. Neither graph passes the vertical line test  
(a)

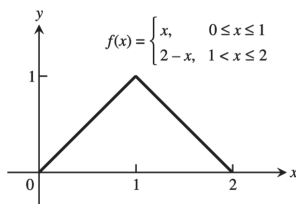


- (b)

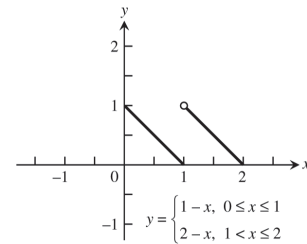


$$|x+y|=1 \Leftrightarrow \begin{cases} x+y=1 \\ \text{or} \\ x+y=-1 \end{cases} \Leftrightarrow \begin{cases} y=1-x \\ \text{or} \\ y=-1-x \end{cases}$$

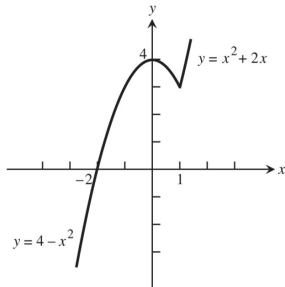
x	0	1	2
y	0	1	0



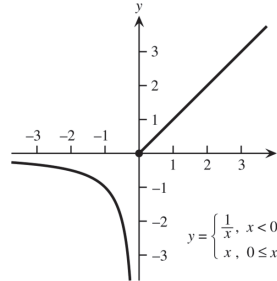
x	0	1	2
y	1	0	0



27.  $F(x) = \begin{cases} 4-x^2, & x \leq 1 \\ x^2+2x, & x > 1 \end{cases}$



28.  $G(x) = \begin{cases} \frac{1}{x}, & x < 0 \\ x, & 0 \leq x \end{cases}$



29. (a) Line through (0, 0) and (1, 1):  $y = x$ ; Line through (1, 1) and (2, 0):  $y = -x + 2$

$$f(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ -x+2, & 1 < x \leq 2 \end{cases}$$

(b)  $f(x) = \begin{cases} 2, & 0 \leq x < 1 \\ 0, & 1 \leq x < 2 \\ 2, & 2 \leq x < 3 \\ 0, & 3 \leq x \leq 4 \end{cases}$

30. (a) Line through (0, 2) and (2, 0):  $y = -x + 2$

Line through (2, 1) and (5, 0):  $m = \frac{0-1}{5-2} = -\frac{1}{3}$ , so  $y = -\frac{1}{3}(x-2) + 1 = -\frac{1}{3}x + \frac{5}{3}$

$$f(x) = \begin{cases} -x+2, & 0 < x \leq 2 \\ -\frac{1}{3}x + \frac{5}{3}, & 2 < x \leq 5 \end{cases}$$

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(b) Line through  $(-1, 0)$  and  $(0, -3)$ :  $m = \frac{-3-0}{0-(-1)} = -3$ , so  $y = -3x - 3$

Line through  $(0, 3)$  and  $(2, -1)$ :  $m = \frac{-1-3}{2-0} = \frac{-4}{2} = -2$ , so  $y = -2x + 3$

$$f(x) = \begin{cases} -3x-3, & -1 < x \leq 0 \\ -2x+3, & 0 < x \leq 2 \end{cases}$$

31. (a) Line through  $(-1, 1)$  and  $(0, 0)$ :  $y = -x$

Line through  $(0, 1)$  and  $(1, 1)$ :  $y = 1$

Line through  $(1, 1)$  and  $(3, 0)$ :  $m = \frac{0-1}{3-1} = \frac{-1}{2} = -\frac{1}{2}$ , so  $y = -\frac{1}{2}(x-1) + 1 = -\frac{1}{2}x + \frac{3}{2}$

$$f(x) = \begin{cases} -x & -1 \leq x < 0 \\ 1 & 0 < x \leq 1 \\ -\frac{1}{2}x + \frac{3}{2} & 1 < x < 3 \end{cases}$$

(b) Line through  $(-2, -1)$  and  $(0, 0)$ :  $y = \frac{1}{2}x$

Line through  $(0, 2)$  and  $(1, 0)$ :  $y = -2x + 2$

Line through  $(1, -1)$  and  $(3, -1)$ :  $y = -1$

$$f(x) = \begin{cases} \frac{1}{2}x & -2 \leq x \leq 0 \\ -2x+2 & 0 < x \leq 1 \\ -1 & 1 < x \leq 3 \end{cases}$$

32. (a) Line through  $(\frac{T}{2}, 0)$  and  $(T, 1)$ :  $m = \frac{1-0}{T-(T/2)} = \frac{2}{T}$ , so  $y = \frac{2}{T}(x - \frac{T}{2}) + 0 = \frac{2}{T}x - 1$

$$f(x) = \begin{cases} 0, & 0 \leq x \leq \frac{T}{2} \\ \frac{2}{T}x - 1, & \frac{T}{2} < x \leq T \end{cases}$$

$$(b) f(x) = \begin{cases} A, & 0 \leq x < \frac{T}{2} \\ -A, & \frac{T}{2} \leq x < T \\ A, & T \leq x < \frac{3T}{2} \\ -A, & \frac{3T}{2} \leq x \leq 2T \end{cases}$$

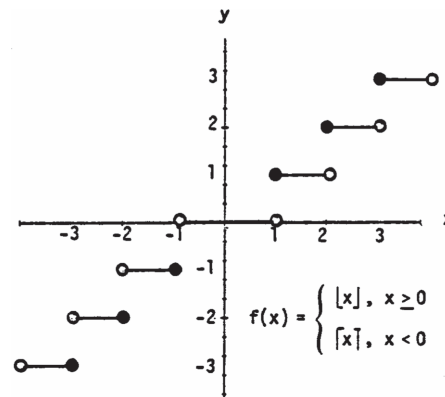
33. (a)  $\lfloor x \rfloor = 0$  for  $x \in [0, 1)$

(b)  $\lceil x \rceil = 0$  for  $x \in (-1, 0]$

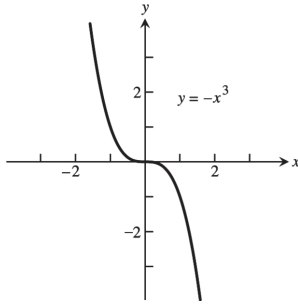
34.  $\lfloor x \rfloor = \lceil x \rceil$  only when  $x$  is an integer.

35. For any real number  $x$ ,  $n \leq x \leq n+1$ , where  $n$  is an integer. Now:  $n \leq x \leq n+1 \Rightarrow -(n+1) \leq -x \leq -n$ .  
By definition:  $\lceil -x \rceil = -n$  and  $\lfloor x \rfloor = n \Rightarrow -\lfloor x \rfloor = -n$ . So  $\lceil -x \rceil = -\lfloor x \rfloor$  for all real  $x$ .

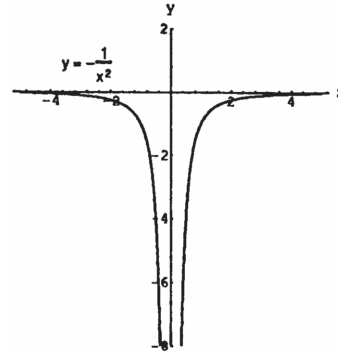
36. To find  $f(x)$  you delete the decimal or fractional portion of  $x$ , leaving only the integer part.



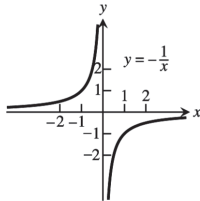
37. Symmetric about the origin  
 Dec:  $-\infty < x < \infty$   
 Inc: nowhere



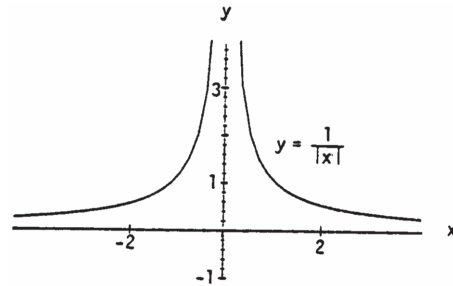
38. Symmetric about the y-axis  
 Dec:  $-\infty < x < 0$   
 Inc:  $0 < x < \infty$



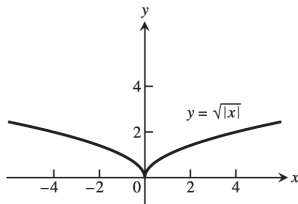
39. Symmetric about the origin  
 Dec: nowhere  
 Inc:  $-\infty < x < 0$   
 $0 < x < \infty$



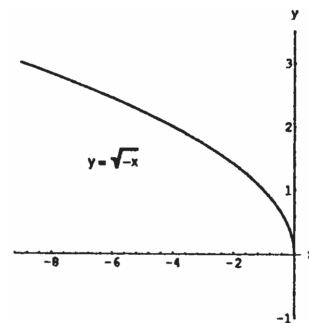
40. Symmetric about the y-axis  
 Dec:  $0 < x < \infty$   
 Inc:  $-\infty < x < 0$



41. Symmetric about the y-axis  
 Dec:  $-\infty < x \leq 0$   
 Inc:  $0 \leq x < \infty$

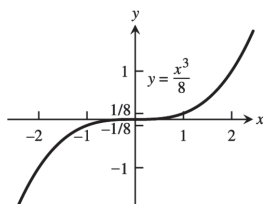


42. No symmetry  
 Dec:  $-\infty < x \leq 0$   
 Inc: nowhere

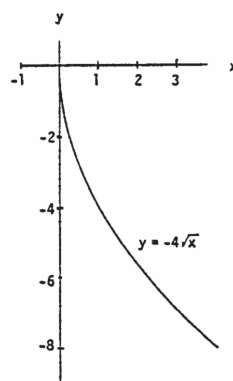


6 Chapter 1 Functions

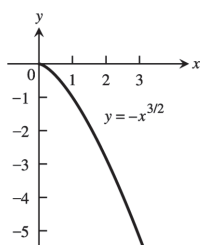
43. Symmetric about the origin  
Dec: nowhere  
Inc:  $-\infty < x < \infty$



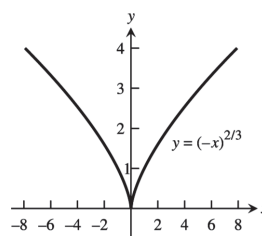
44. No symmetry  
Dec:  $0 \leq x < \infty$   
Inc: nowhere



45. No symmetry  
Dec:  $0 \leq x < \infty$   
Inc: nowhere



46. Symmetric about the y-axis  
Dec:  $-\infty < x \leq 0$   
Inc:  $0 \leq x < \infty$



47. Since a horizontal line not through the origin is symmetric with respect to the y-axis, but not with respect to the origin, the function is even.

48.  $f(x) = x^{-5} = \frac{1}{x^5}$  and  $f(-x) = (-x)^{-5} = \frac{1}{(-x)^5} = -\left(\frac{1}{x^5}\right) = -f(x)$ . Thus the function is odd.

49. Since  $f(x) = x^2 + 1 = (-x)^2 + 1 = f(-x)$ . The function is even.

50. Since  $[f(x) = x^2 + x] \neq [f(-x) = (-x)^2 - x]$  and  $[f(x) = x^2 + x] \neq [-f(x) = -(x)^2 - x]$  the function is neither even nor odd.

51. Since  $g(x) = x^3 + x$ ,  $g(-x) = -x^3 - x = -(x^3 + x) = -g(x)$ . So the function is odd.

52.  $g(x) = x^4 + 3x^2 - 1 = (-x)^4 + 3(-x)^2 - 1 = g(-x)$ , thus the function is even.

53.  $g(x) = \frac{1}{x^2 - 1} = \frac{1}{(-x)^2 - 1} = g(-x)$ . Thus the function is even.

54.  $g(x) = \frac{x}{x^2 - 1}$ ;  $g(-x) = -\frac{x}{x^2 - 1} = -g(x)$ . So the function is odd.

55.  $h(t) = \frac{1}{t - 1}$ ;  $h(-t) = \frac{1}{-t - 1}$ ;  $-h(t) = \frac{1}{1 - t}$ . Since  $h(t) \neq -h(t)$  and  $h(t) \neq h(-t)$ , the function is neither even nor odd.

56. Since  $|t^3| = |(-t)^3|$ ,  $h(t) = h(-t)$  and the function is even.
57.  $h(t) = 2t + 1$ ,  $h(-t) = -2t + 1$ . So  $h(t) \neq h(-t)$ .  $-h(t) = -2t - 1$ , so  $h(t) \neq -h(t)$ . The function is neither even nor odd.
58.  $h(t) = 2|t| + 1$  and  $h(-t) = 2|-t| + 1 = 2|t| + 1$ . So  $h(t) = h(-t)$  and the function is even.
59.  $g(x) = \sin 2x$ ;  $g(-x) = -\sin 2x = -g(x)$ . So the function is odd.
60.  $g(x) = \sin x^2$ ;  $g(-x) = \sin x^2 = g(x)$ . So the function is even.
61.  $g(x) = \cos 3x$ ;  $g(-x) = \cos 3x = g(x)$ . So the function is even.
62.  $g(x) = 1 + \cos x$ ;  $g(-x) = 1 + \cos x = g(x)$ . So the function is even.
63.  $s = kt \Rightarrow 25 = k(75) \Rightarrow k = \frac{1}{3} \Rightarrow s = \frac{1}{3}t$ ;  $60 = \frac{1}{3}t \Rightarrow t = 180$
64.  $K = cv^2 \Rightarrow 12960 = c(18)^2 \Rightarrow c = 40 \Rightarrow K = 40v^2$ ;  $K = 40(10)^2 = 4000$  joules
65.  $r = \frac{k}{s} \Rightarrow 6 = \frac{k}{4} \Rightarrow k = 24 \Rightarrow r = \frac{24}{s}$ ;  $10 = \frac{24}{s} \Rightarrow s = \frac{12}{5}$
66.  $P = \frac{k}{V} \Rightarrow 14.7 = \frac{k}{1000} \Rightarrow k = 14700 \Rightarrow P = \frac{14700}{V}$ ;  $23.4 = \frac{14700}{V} \Rightarrow V = \frac{24500}{39} \approx 628.2 \text{ in}^3$
67.  $V = f(x) = x(14 - 2x)(22 - 2x) = 4x^3 - 72x^2 + 308x$ ;  $0 < x < 7$ .
68. (a) Let  $h$  = height of the triangle. Since the triangle is isosceles,  $(\overline{AB})^2 + (\overline{AB})^2 = 2^2 \Rightarrow \overline{AB} = \sqrt{2}$ . So,  
 $h^2 + 1^2 = (\sqrt{2})^2 \Rightarrow h = 1 \Rightarrow B$  is at  $(0, 1) \Rightarrow$  slope of  $AB = -1 \Rightarrow$  The equation of  $AB$  is  
 $y = f(x) = -x + 1$ ;  $x \in [0, 1]$ .
- (b)  $A(x) = 2xy = 2x(-x + 1) = -2x^2 + 2x$ ;  $x \in [0, 1]$ .
69. (a) Graph  $h$  because it is an even function and rises less rapidly than does Graph  $g$ .  
 (b) Graph  $f$  because it is an odd function.  
 (c) Graph  $g$  because it is an even function and rises more rapidly than does Graph  $h$ .
70. (a) Graph  $f$  because it is linear.  
 (b) Graph  $g$  because it contains  $(0, 1)$ .  
 (c) Graph  $h$  because it is a nonlinear odd function.

71. (a) From the graph,  $\frac{x}{2} > 1 + \frac{4}{x} \Rightarrow x \in (-2, 0) \cup (4, \infty)$

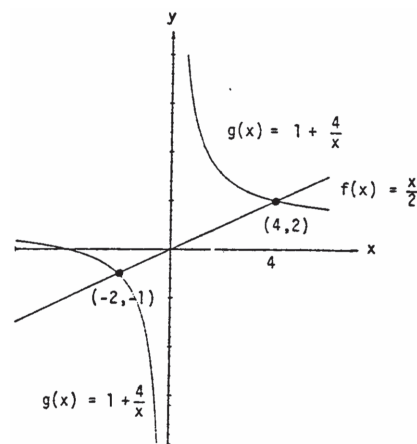
(b)  $\frac{x}{2} > 1 + \frac{4}{x} \Rightarrow \frac{x}{2} - 1 - \frac{4}{x} > 0$

$x > 0: \frac{x}{2} - 1 - \frac{4}{x} > 0 \Rightarrow \frac{x^2 - 2x - 8}{2x} > 0 \Rightarrow \frac{(x-4)(x+2)}{2x} > 0$   
 $\Rightarrow x > 4$  since  $x$  is positive;

$x < 0: \frac{x}{2} - 1 - \frac{4}{x} > 0 \Rightarrow \frac{x^2 - 2x - 8}{2x} < 0 \Rightarrow \frac{(x-4)(x+2)}{2x} < 0$   
 $\Rightarrow x < -2$  since  $x$  is negative;  
 sign of  $(x-4)(x+2)$



Solution interval:  $(-2, 0) \cup (4, \infty)$



72. (a) From the graph,  $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow x \in (-\infty, -5) \cup (-1, 1)$

(b) Case  $x < -1: \frac{3}{x-1} < \frac{2}{x+1} \Rightarrow \frac{3(x+1)}{x-1} > 2$

$\Rightarrow 3x + 3 < 2x - 2 \Rightarrow x < -5.$

Thus,  $x \in (-\infty, -5)$  solves the inequality.

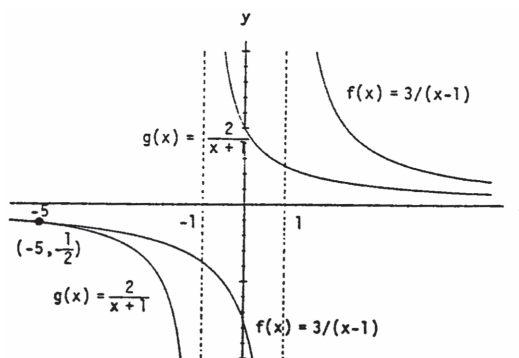
Case  $-1 < x < 1: \frac{3}{x-1} < \frac{2}{x+1} \Rightarrow \frac{3(x+1)}{x-1} < 2$

$\Rightarrow 3x + 3 > 2x - 2 \Rightarrow x > -5$  which  
 is true if  $x > -1$ . Thus,  $x \in (-1, 1)$   
 solves the inequality.

Case  $1 < x: \frac{3}{x-1} < \frac{2}{x+1} \Rightarrow 3x + 3 < 2x - 2 \Rightarrow x < -5$

which is never true if  $1 < x$ ,  
 so no solution here.

In conclusion,  $x \in (-\infty, -5) \cup (-1, 1)$ .



73. A curve symmetric about the  $x$ -axis will not pass the vertical line test because the points  $(x, y)$  and  $(x, -y)$  lie on the same vertical line. The graph of the function  $y = f(x) = 0$  is the  $x$ -axis, a horizontal line for which there is a single  $y$ -value, 0, for any  $x$ .

74. price =  $40 + 5x$ , quantity =  $300 - 25x \Rightarrow R(x) = (40 + 5x)(300 - 25x)$

75.  $x^2 + x^2 = h^2 \Rightarrow x = \frac{h}{\sqrt{2}} = \frac{\sqrt{2}h}{2}$ ; cost =  $5(2x) + 10h \Rightarrow C(h) = 10\left(\frac{\sqrt{2}h}{2}\right) + 10h = 5h(\sqrt{2} + 2)$

76. (a) Note that 2 mi = 10,560 ft, so there are  $\sqrt{800^2 + x^2}$  feet of river cable at \$180 per foot and  $(10,560 - x)$  feet of land cable at \$100 per foot. The cost is  $C(x) = 180\sqrt{800^2 + x^2} + 100(10,560 - x)$ .

(b)  $C(0) = \$1,200,000$

$C(500) \approx \$1,175,812$

$C(1000) \approx \$1,186,512$

$C(1500) \approx \$1,212,000$

$C(2000) \approx \$1,243,732$

$C(2500) \approx \$1,278,479$

$C(3000) \approx \$1,314,870$

Values beyond this are all larger. It would appear that the least expensive location is less than 2000 feet from the point  $P$ .



**1.2 COMBINING FUNCTIONS; SHIFTING AND SCALING GRAPHS**

1.  $D_f: -\infty < x < \infty, D_g: x \geq 1 \Rightarrow D_{f+g} = D_{fg}: x \geq 1. R_f: -\infty < y < \infty, R_g: y \geq 0, R_{f+g}: y \geq 1, R_{fg}: y \geq 0$
2.  $D_f: x+1 \geq 0 \Rightarrow x \geq -1, D_g: x-1 \geq 0 \Rightarrow x \geq 1. \text{ Therefore } D_{f+g} = D_{fg}: x \geq 1.$   
 $R_f = R_g: y \geq 0, R_{f+g}: y \geq \sqrt{2}, R_{fg}: y \geq 0$
3.  $D_f: -\infty < x < \infty, D_g: -\infty < x < \infty, D_{f/g}: -\infty < x < \infty, D_{g/f}: -\infty < x < \infty, R_f: y = 2, R_g: y \geq 1, R_{f/g}: 0 < y \leq 2,$   
 $R_{g/f}: \frac{1}{2} \leq y < \infty$
4.  $D_f: -\infty < x < \infty, D_g: x \geq 0, D_{f/g}: x \geq 0, D_{g/f}: x \geq 0; R_f: y = 1, R_g: y \geq 1, R_{f/g}: 0 < y \leq 1, R_{g/f}: 1 \leq y < \infty$
5. (a) 2 (b) 22 (c)  $x^2 + 2$   
 (d)  $(x+5)^2 - 3 = x^2 + 10x + 22$  (e) 5 (f) -2  
 (g)  $x+10$  (h)  $(x^2 - 3)^2 - 3 = x^4 - 6x^2 + 6$
6. (a)  $-\frac{1}{3}$  (b) 2 (c)  $\frac{1}{x+1} - 1 = \frac{-x}{x+1}$   
 (d)  $\frac{1}{x}$  (e) 0 (f)  $\frac{3}{4}$   
 (g)  $x-2$  (h)  $\frac{1}{\frac{1}{x+1} + 1} = \frac{1}{\frac{x+2}{x+1}} = \frac{x+1}{x+2}$
7.  $(f \circ g \circ h)(x) = f(g(h(x))) = f(g(4-x)) = f(3(4-x)) = f(12-3x) = (12-3x) + 1 = 13-3x$
8.  $(f \circ g \circ h)(x) = f(g(h(x))) = f(g(x^2)) = f(2(x^2)-1) = f(2x^2-1) = 3(2x^2-1) + 4 = 6x^2 + 1$
9.  $(f \circ g \circ h)(x) = f(g(h(x))) = f\left(g\left(\frac{1}{x}\right)\right) = f\left(\frac{1}{\frac{1}{x} + 4}\right) = f\left(\frac{x}{1+4x}\right) = \sqrt{\frac{x}{1+4x} + 1} = \sqrt{\frac{5x+1}{1+4x}}$
10.  $(f \circ g \circ h)(x) = f(g(h(x))) = f\left(g\left(\sqrt{2-x}\right)\right) = f\left(\frac{(\sqrt{2-x})^2}{(\sqrt{2-x})^2 + 1}\right) = f\left(\frac{2-x}{3-x}\right) = \frac{\frac{2-x}{3-x} + 2}{3 - \frac{2-x}{3-x}} = \frac{8-3x}{7-2x}$
11. (a)  $(f \circ g)(x)$  (b)  $(j \circ g)(x)$  (c)  $(g \circ g)(x)$   
 (d)  $(j \circ j)(x)$  (e)  $(g \circ h \circ f)(x)$  (f)  $(h \circ j \circ f)(x)$
12. (a)  $(f \circ j)(x)$  (b)  $(g \circ h)(x)$  (c)  $(h \circ h)(x)$   
 (d)  $(f \circ f)(x)$  (e)  $(j \circ g \circ f)(x)$  (f)  $(g \circ f \circ h)(x)$
13.
 

g(x)	f(x)	(f ∘ g)(x)
(a) $x-7$	$\sqrt{x}$	$\sqrt{x-7}$
(b) $x+2$	$3x$	$3(x+2) = 3x+6$
(c) $x^2$	$\sqrt{x-5}$	$\sqrt{x^2-5}$
(d) $\frac{x}{x-1}$	$\frac{x}{x-1}$	$\frac{\frac{x}{x-1}}{\frac{x}{x-1} - 1} = \frac{x}{x - (x-1)} = x$
(e) $\frac{1}{x-1}$	$1 + \frac{1}{x}$	$x$

(f)  $\frac{1}{x} \quad \frac{1}{x} \quad x$

14. (a)  $(f \circ g)(x) = |g(x)| = \frac{1}{|x-1|}$ .

(b)  $(f \circ g)(x) = \frac{g(x)-1}{g(x)} = \frac{x}{x+1} \Rightarrow 1 - \frac{1}{g(x)} = \frac{x}{x+1} \Rightarrow 1 - \frac{x}{x+1} = \frac{1}{g(x)} \Rightarrow \frac{1}{x+1} = \frac{1}{g(x)}$ , so  $g(x) = x+1$ .

(c) Since  $(f \circ g)(x) = \sqrt{g(x)} = |x|$ ,  $g(x) = x^2$ .

(d) Since  $(f \circ g)(x) = f(\sqrt{x}) = |x|$ ,  $f(x) = x^2$ . (Note that the domain of the composition is  $[0, \infty)$ .)

The completed table is shown. Note that the absolute value sign in part (d) is optional.

$g(x)$	$f(x)$	$(f \circ g)(x)$
$\frac{1}{x-1}$	$ x $	$\frac{1}{ x-1 }$
$x+1$	$\frac{x-1}{x}$	$\frac{x}{x+1}$
$x^2$	$\sqrt{x}$	$ x $
$\sqrt{x}$	$x^2$	$ x $

15. (a)  $f(g(-1)) = f(1) = 1$       (b)  $g(f(0)) = g(-2) = 2$       (c)  $f(f(-1)) = f(0) = -2$   
 (d)  $g(g(2)) = g(0) = 0$       (e)  $g(f(-2)) = g(1) = -1$       (f)  $f(g(1)) = f(-1) = 0$

16. (a)  $f(g(0)) = f(-1) = 2 - (-1) = 3$ , where  $g(0) = 0 - 1 = -1$   
 (b)  $g(f(3)) = g(-1) = -(-1) = 1$ , where  $f(3) = 2 - 3 = -1$   
 (c)  $g(g(-1)) = g(1) = 1 - 1 = 0$ , where  $g(-1) = -(-1) = 1$   
 (d)  $f(f(2)) = f(0) = 2 - 0 = 2$ , where  $f(2) = 2 - 2 = 0$   
 (e)  $g(f(0)) = g(2) = 2 - 1 = 1$ , where  $f(0) = 2 - 0 = 2$   
 (f)  $f\left(g\left(\frac{1}{2}\right)\right) = f\left(-\frac{1}{2}\right) = 2 - \left(-\frac{1}{2}\right) = \frac{5}{2}$ , where  $g\left(\frac{1}{2}\right) = \frac{1}{2} - 1 = -\frac{1}{2}$

17. (a)  $(f \circ g)(x) = f(g(x)) = \sqrt{\frac{1}{x} + 1} = \sqrt{\frac{1+x}{x}}$   
 $(g \circ f)(x) = g(f(x)) = \frac{1}{\sqrt{x+1}}$   
 (b) Domain  $(f \circ g)$ :  $(-\infty, -1] \cup (0, \infty)$ , domain  $(g \circ f)$ :  $(-1, \infty)$   
 (c) Range  $(f \circ g)$ :  $(1, \infty)$ , range  $(g \circ f)$ :  $(0, \infty)$

18. (a)  $(f \circ g)(x) = f(g(x)) = 1 - 2\sqrt{x} + x$   
 $(g \circ f)(x) = g(f(x)) = 1 - |x|$   
 (b) Domain  $(f \circ g)$ :  $[0, \infty)$ , domain  $(g \circ f)$ :  $(-\infty, \infty)$   
 (c) Range  $(f \circ g)$ :  $(0, \infty)$ , range  $(g \circ f)$ :  $(-\infty, 1]$

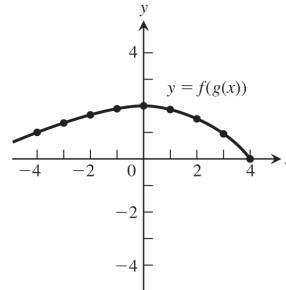
19.  $(f \circ g)(x) = x \Rightarrow f(g(x)) = x \Rightarrow \frac{g(x)}{g(x)-2} = x \Rightarrow g(x) = (g(x)-2)x = x \cdot g(x) - 2x$   
 $\Rightarrow g(x) - x \cdot g(x) = -2x \Rightarrow g(x) = -\frac{2x}{1-x} = \frac{2x}{x-1}$

20.  $(f \circ g)(x) = x+2 \Rightarrow f(g(x)) = x+2 \Rightarrow 2(g(x))^3 - 4 = x+2 \Rightarrow (g(x))^3 = \frac{x+6}{2} \Rightarrow g(x) = \sqrt[3]{\frac{x+6}{2}}$

21.  $V = V(s) = V(s(t)) = V(2t-3)$   
 $= (2t-3)^2 + 2(2t-3) + 3$   
 $= 4t^2 - 8t + 6$

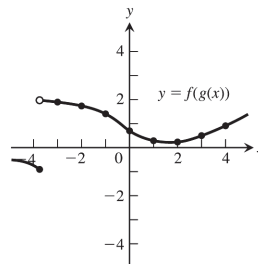
22. (a)

$x$	-4	-3	-2	-1	0	1	2	3	4
$g(x)$	-2	-1	-0.5	-0.2	0	0.2	0.5	1	2
$f(g(x))$	1	1.3	1.6	1.8	2	1.8	1.5	1	0



(b)

$x$	-4	-3	-2	-1	0	1	2	3	4
$g(x)$	1.5	0.3	-0.7	-1.5	-2.4	-2.8	-3	-2.7	-2
$f(g(x))$	-0.8	1.9	1.7	1.5	0.7	0.3	0.2	0.5	0.9



23. (a)  $y = -(x + 7)^2$

(b)  $y = -(x - 4)^2$

24. (a)  $y = x^2 + 3$

(b)  $y = x^2 - 5$

25. (a) Position 4

(b) Position 1

(c) Position 2

(d) Position 3

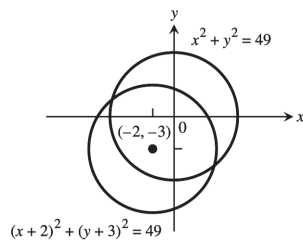
26. (a)  $y = -(x - 1)^2 + 4$

(b)  $y = -(x + 2)^2 + 3$

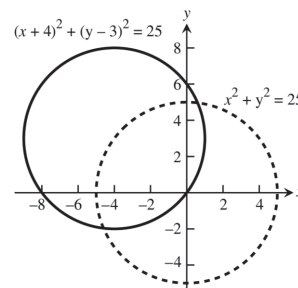
(c)  $y = -(x + 4)^2 - 1$

(d)  $y = -(x - 2)^2$

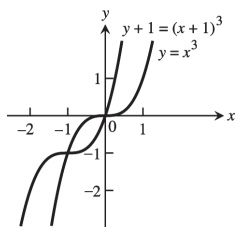
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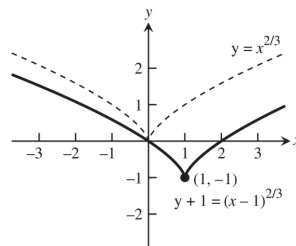
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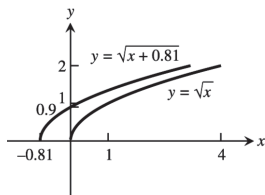
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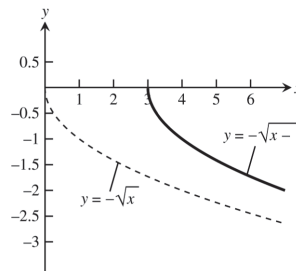
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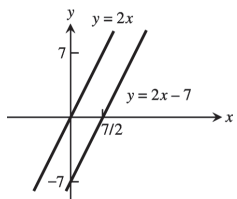
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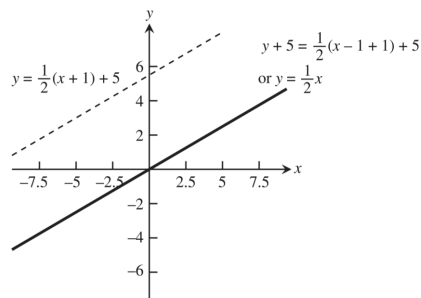
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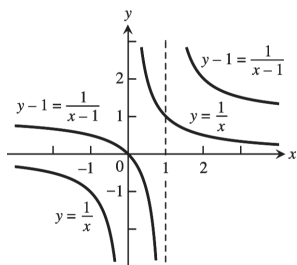
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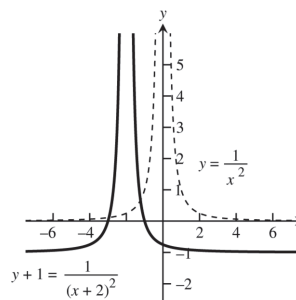
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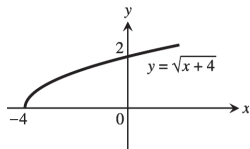
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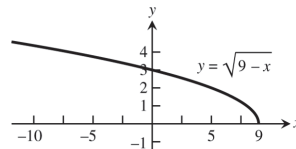
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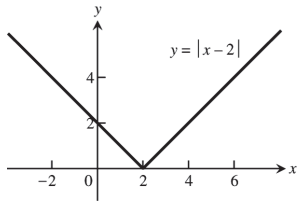
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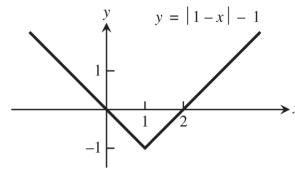
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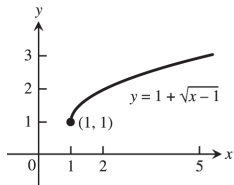
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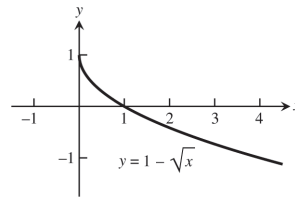
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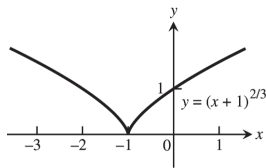
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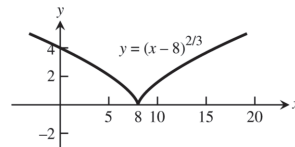
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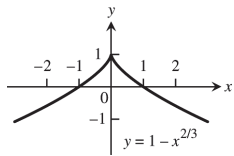
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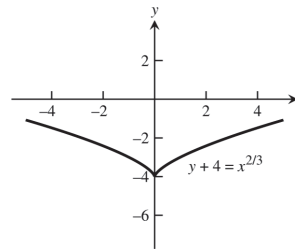
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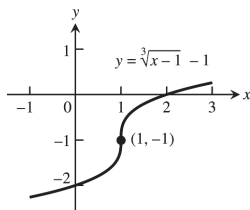
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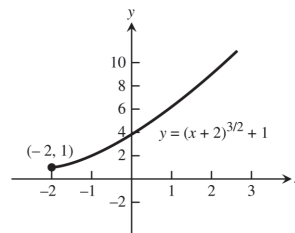
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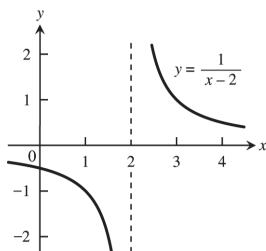
47.



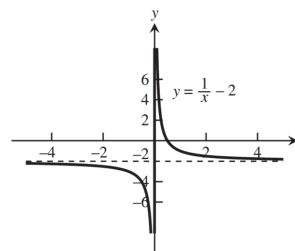
48.



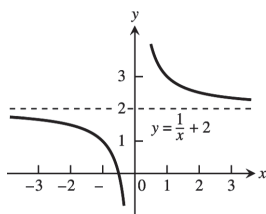
49.



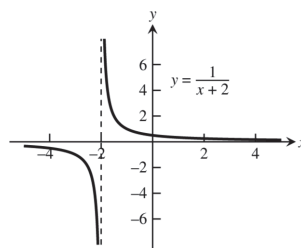
50.



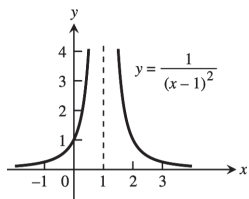
51.



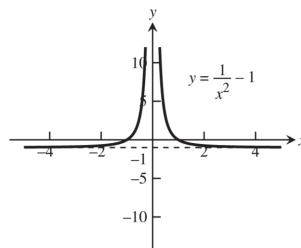
52.



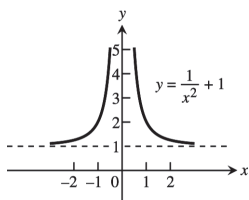
53.



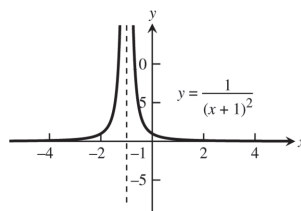
54.



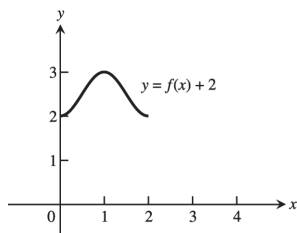
55.



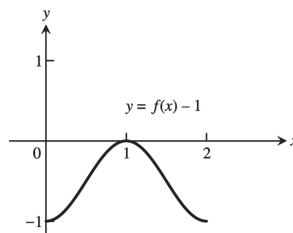
56.



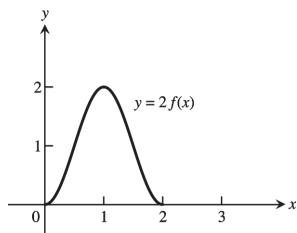
57. (a) domain:  $[0, 2]$ ; range:  $[2, 3]$



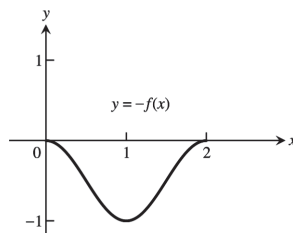
(b) domain:  $[0, 2]$ ; range:  $[-1, 0]$



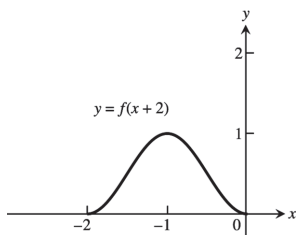
(c) domain:  $[0, 2]$ ; range:  $[0, 2]$



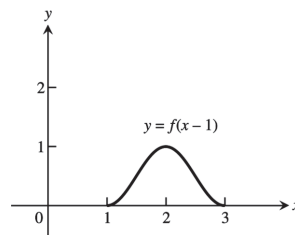
(d) domain:  $[0, 2]$ ; range:  $[-1, 0]$



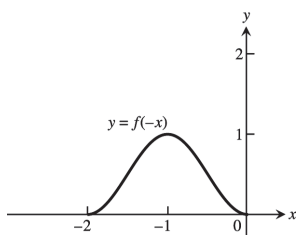
(e) domain:  $[-2, 0]$ ; range:  $[0, 1]$



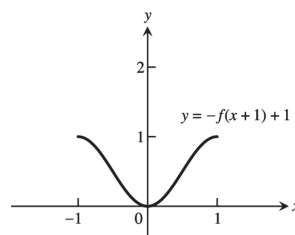
(f) domain:  $[1, 3]$ ; range:  $[0, 1]$



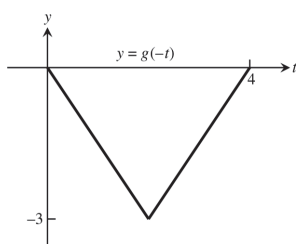
(g) domain:  $[-2, 0]$ ; range:  $[0, 1]$



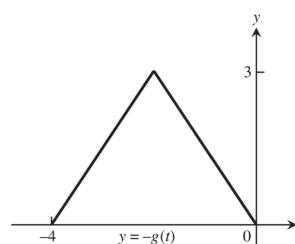
(h) domain:  $[-1, 1]$ ; range:  $[0, 1]$



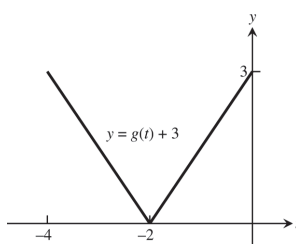
58. (a) domain:  $[0, 4]$ ; range:  $[-3, 0]$



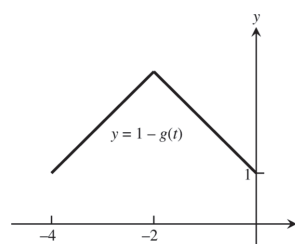
(b) domain:  $[-4, 0]$ ; range:  $[0, 3]$



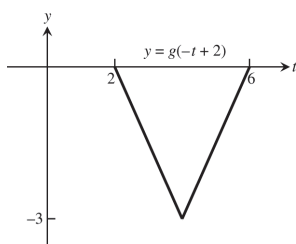
(c) domain:  $[-4, 0]$ ; range:  $[0, 3]$



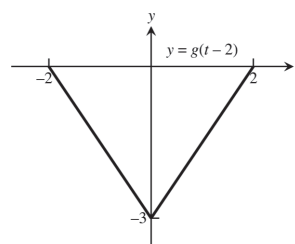
(d) domain:  $[-4, 0]$ ; range:  $[1, 4]$



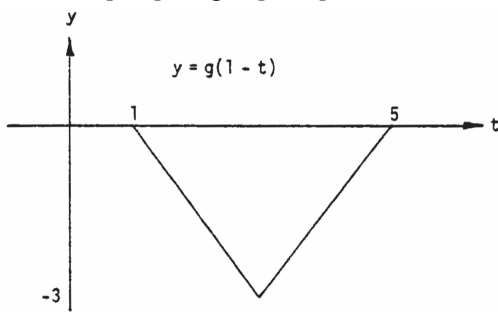
(e) domain:  $[2, 4]$ ; range:  $[-3, 0]$



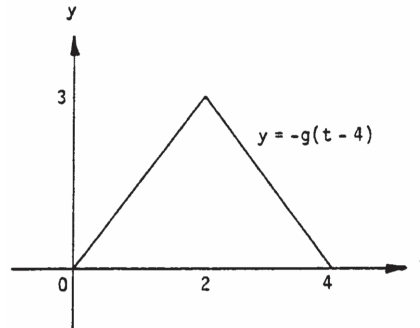
(f) domain:  $[-2, 2]$ ; range:  $[-3, 0]$



(g) domain:  $[1, 5]$ ; range:  $[-3, 0]$



(h) domain:  $[0, 4]$ ; range:  $[0, 3]$



59.  $y = 3x^2 - 3$

60.  $y = (2x)^2 - 1 = 4x^2 - 1$

61.  $y = \frac{1}{2}\left(1 + \frac{1}{x^2}\right) = \frac{1}{2} + \frac{1}{2x^2}$

62.  $y = 1 + \frac{1}{(x/3)^2} = 1 + \frac{9}{x^2}$

63.  $y = \sqrt{4x+1}$

64.  $y = 3\sqrt{x+1}$

65.  $y = \sqrt{4 - \left(\frac{x}{2}\right)^2} = \frac{1}{2}\sqrt{16 - x^2}$

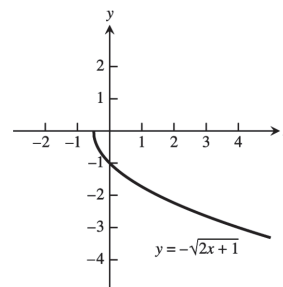
66.  $y = \frac{1}{3}\sqrt{4 - x^2}$

67.  $y = 1 - (3x)^3 = 1 - 27x^3$

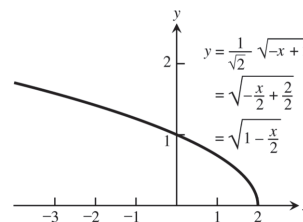
68.  $y = 1 - \left(\frac{x}{2}\right)^3 = 1 - \frac{x^3}{8}$

69. Let  $y = -\sqrt{2x+1} = f(x)$  and let  $g(x) = x^{1/2}$ ,  
 $h(x) = \left(x + \frac{1}{2}\right)^{1/2}$ ,  $i(x) = \sqrt{2}\left(x + \frac{1}{2}\right)^{1/2}$ , and  
 $j(x) = -\left[\sqrt{2}\left(x + \frac{1}{2}\right)^{1/2}\right] = f(x)$ . The graph of  $h(x)$

is the graph of  $g(x)$  shifted left  $\frac{1}{2}$  unit; the graph of  $i(x)$  is the graph of  $h(x)$  stretched vertically by a factor of  $\sqrt{2}$ ; and the graph of  $j(x) = f(x)$  is the graph of  $i(x)$  reflected across the  $x$ -axis.

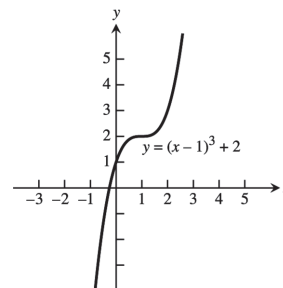


70. Let  $y = \sqrt{1 - \frac{x}{2}} = f(x)$ . Let  $g(x) = (-x)^{1/2}$ ,  
 $h(x) = (-x+2)^{1/2}$ , and  $i(x) = \frac{1}{\sqrt{2}}(-x+2)^{1/2} =$   
 $\sqrt{1 - \frac{x}{2}} = f(x)$ . The graph of  $g(x)$  is the graph of  $y = \sqrt{x}$  reflected across the  $x$ -axis. The graph of  $h(x)$  is the graph of  $g(x)$  shifted right two units. And the graph of  $i(x)$  is the graph of  $h(x)$  compressed vertically by a factor of  $\sqrt{2}$ .



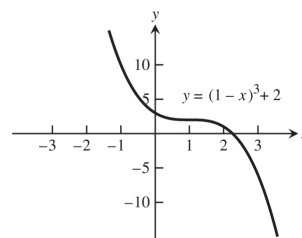


71.  $y = f(x) = x^3$ . Shift  $f(x)$  one unit right followed by a shift two units up to get  $g(x) = (x-1)^3 + 2$ .

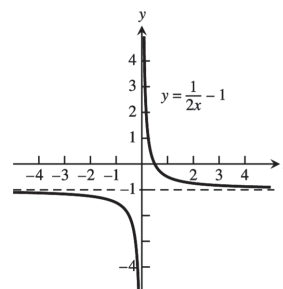


72.  $y = (1-x)^3 + 2 = -[(x-1)^3 + (-2)] = f(x)$ .

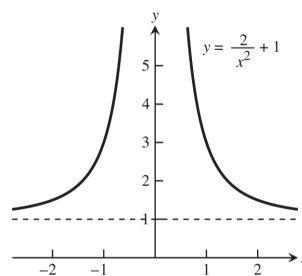
Let  $g(x) = x^3$ ,  $h(x) = (x-1)^3$ ,  $i(x) = (x-1)^3 + (-2)$ , and  $j(x) = -[(x-1)^3 + (-2)]$ . The graph of  $h(x)$  is the graph of  $g(x)$  shifted right one unit; the graph of  $i(x)$  is the graph of  $h(x)$  shifted down two units; and the graph of  $f(x)$  is the graph of  $i(x)$  reflected across the  $x$ -axis.



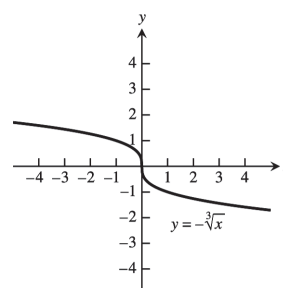
73. Compress the graph of  $f(x) = \frac{1}{x}$  horizontally by a factor of 2 to get  $g(x) = \frac{1}{2x}$ . Then shift  $g(x)$  vertically down 1 unit to get  $h(x) = \frac{1}{2x} - 1$ .



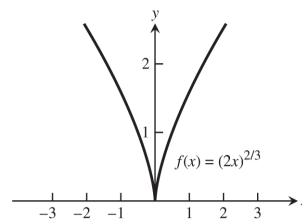
74. Let  $f(x) = \frac{1}{x^2}$  and  $g(x) = \frac{2}{x^2} + 1 = \frac{1}{\left(\frac{x}{\sqrt{2}}\right)^2} + 1$   
 $= \frac{1}{(x/\sqrt{2})^2} + 1 = \frac{1}{[(1/\sqrt{2})x]^2} + 1$ . Since  $\sqrt{2} \approx 1.4$ , we see that the graph of  $f(x)$  stretched horizontally by a factor of 1.4 and shifted up 1 unit is the graph of  $g(x)$ .



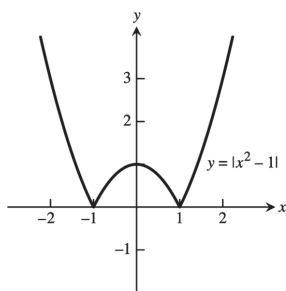
75. Reflect the graph of  $y = f(x) = \sqrt[3]{x}$  across the  $x$ -axis to get  $g(x) = -\sqrt[3]{x}$ .



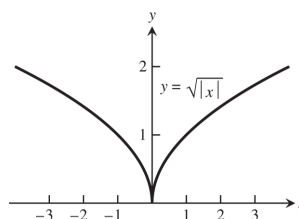
76.  $y = f(x) = (-2x)^{2/3} = [(-1)(2x)]^{2/3} = (-1)^{2/3} (2x)^{2/3} = (2x)^{2/3}$ . So the graph of  $f(x)$  is the graph of  $g(x) = x^{2/3}$  compressed horizontally by a factor of 2.



77.



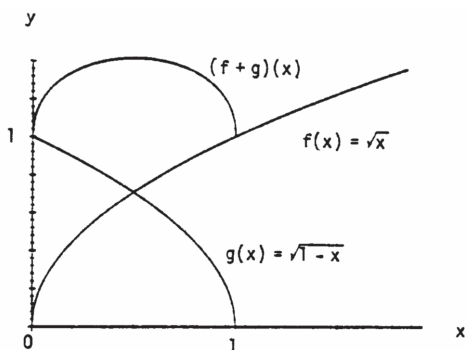
78.



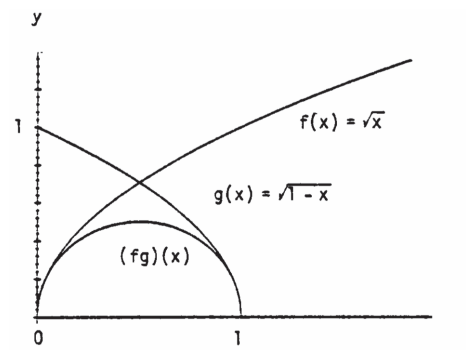
79. (a)  $(fg)(-x) = f(-x)g(-x) = f(x)(-g(x)) = -(fg)(x)$ , odd  
 (b)  $\left(\frac{f}{g}\right)(-x) = \frac{f(-x)}{g(-x)} = \frac{f(x)}{-g(x)} = -\left(\frac{f}{g}\right)(x)$ , odd  
 (c)  $\left(\frac{g}{f}\right)(-x) = \frac{g(-x)}{f(-x)} = \frac{-g(x)}{f(x)} = -\left(\frac{g}{f}\right)(x)$ , odd  
 (d)  $f^2(-x) = f(-x)f(-x) = f(x)f(x) = f^2(x)$ , even  
 (e)  $g^2(-x) = (g(-x))^2 = (-g(x))^2 = g^2(x)$ , even  
 (f)  $(f \circ g)(-x) = f(g(-x)) = f(-g(x)) = f(g(x)) = (f \circ g)(x)$ , even  
 (g)  $(g \circ f)(-x) = g(f(-x)) = g(f(x)) = (g \circ f)(x)$ , even  
 (h)  $(f \circ f)(-x) = f(f(-x)) = f(f(x)) = (f \circ f)(x)$ , even  
 (i)  $(g \circ g)(-x) = g(g(-x)) = g(-g(x)) = -g(g(x)) = -(g \circ g)(x)$ , odd

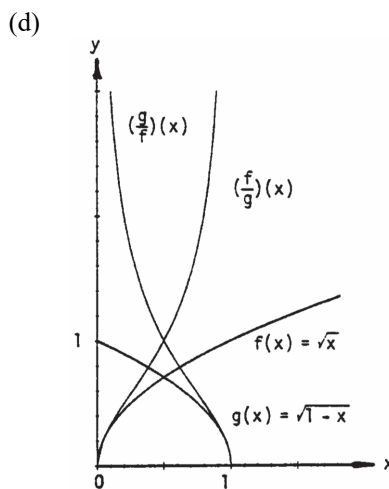
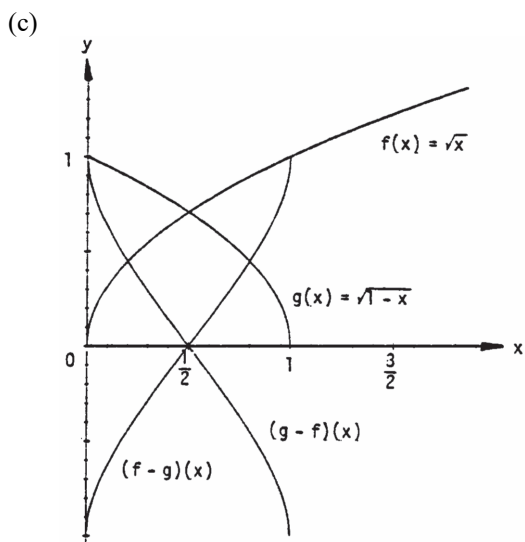
80. Yes,  $f(x) = 0$  is both even and odd since  $f(-x) = 0 = f(x)$  and  $f(-x) = 0 = -f(x)$ .

81. (a)

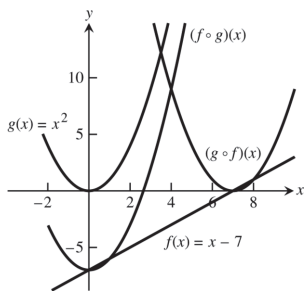


(b)





82.



1.3 TRIGONOMETRIC FUNCTIONS

- (a)  $s = r\theta = (10)\left(\frac{4\pi}{5}\right) = 8\pi$  m                      (b)  $s = r\theta = (10)(110^\circ)\left(\frac{\pi}{180^\circ}\right) = \frac{110\pi}{18} = \frac{55\pi}{9}$  m
- $\theta = \frac{s}{r} = \frac{10\pi}{8} = \frac{5\pi}{4}$  radians and  $\frac{5\pi}{4}\left(\frac{180^\circ}{\pi}\right) = 225^\circ$
- $\theta = 80^\circ \Rightarrow \theta = 80^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{4\pi}{9} \Rightarrow s = (6)\left(\frac{4\pi}{9}\right) = 8.4$  in. (since the diameter = 12 in.  $\Rightarrow$  radius = 6 in.)
- $d = 1$  meter  $\Rightarrow r = 50$  cm  $\Rightarrow \theta = \frac{s}{r} = \frac{30}{50} = 0.6$  rad or  $0.6\left(\frac{180^\circ}{\pi}\right) \approx 34^\circ$

5.

$\theta$	$-\pi$	$-\frac{2\pi}{3}$	0	$\frac{\pi}{2}$	$\frac{3\pi}{4}$
$\sin \theta$	0	$-\frac{\sqrt{3}}{2}$	0	1	$\frac{1}{\sqrt{2}}$
$\cos \theta$	-1	$-\frac{1}{2}$	1	0	$-\frac{1}{\sqrt{2}}$
$\tan \theta$	0	$\sqrt{3}$	0	und.	-1
$\cot \theta$	und.	$\frac{1}{\sqrt{3}}$	und.	0	-1
$\sec \theta$	-1	-2	1	und.	$-\sqrt{2}$
$\csc \theta$	und.	$-\frac{2}{\sqrt{3}}$	und.	1	$\sqrt{2}$

6.

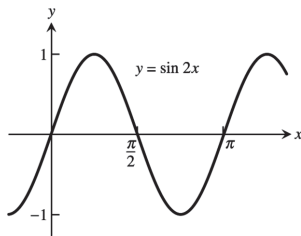
$\theta$	$-\frac{3\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{5\pi}{6}$
$\sin \theta$	1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\cos \theta$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$
$\tan \theta$	und.	$-\sqrt{3}$	$-\frac{1}{\sqrt{3}}$	1	$-\frac{1}{\sqrt{3}}$
$\cot \theta$	0	$-\frac{1}{\sqrt{3}}$	$-\sqrt{3}$	1	$-\sqrt{3}$
$\sec \theta$	und.	2	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	$-\frac{2}{\sqrt{3}}$
$\csc \theta$	1	$-\frac{2}{\sqrt{3}}$	-2	$\sqrt{2}$	2

7.  $\cos x = -\frac{4}{5}, \tan x = -\frac{3}{4}$

9.  $\sin x = -\frac{\sqrt{8}}{3}, \tan x = -\sqrt{8}$

11.  $\sin x = -\frac{1}{\sqrt{5}}, \cos x = -\frac{2}{\sqrt{5}}$

13.



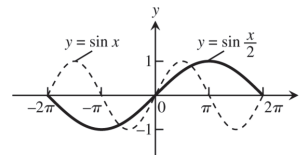
period =  $\pi$

8.  $\sin x = \frac{2}{\sqrt{5}}, \cos x = \frac{1}{\sqrt{5}}$

10.  $\sin x = \frac{12}{13}, \tan x = -\frac{12}{5}$

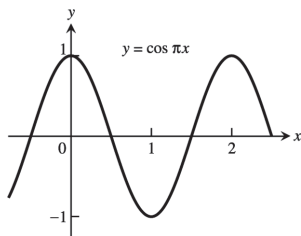
12.  $\cos x = -\frac{\sqrt{3}}{2}, \tan x = \frac{1}{\sqrt{3}}$

14.



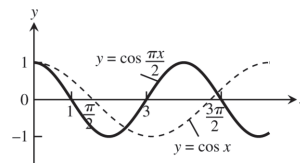
period =  $4\pi$

15.



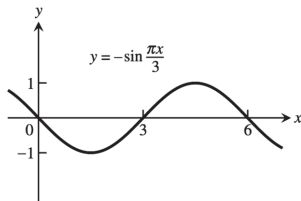
period = 2

16.



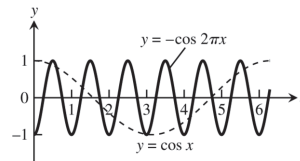
period = 4

17.



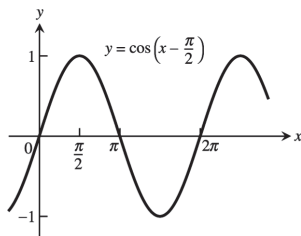
period = 6

18.



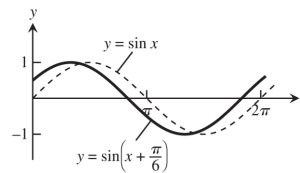
period = 1

19.



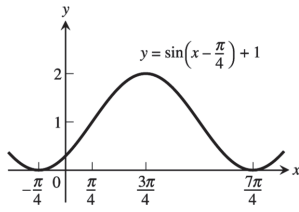
period =  $2\pi$

20.



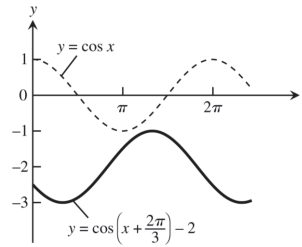
period =  $2\pi$

21.



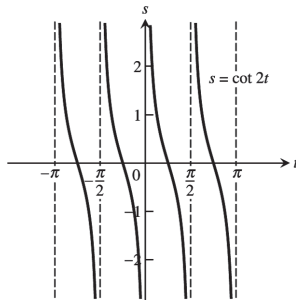
period =  $2\pi$

22.

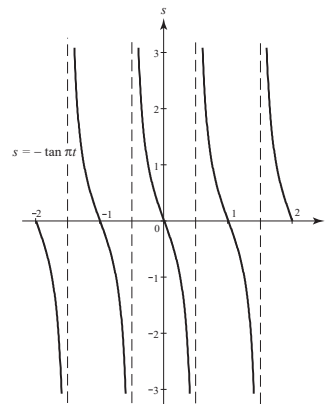


period =  $2\pi$

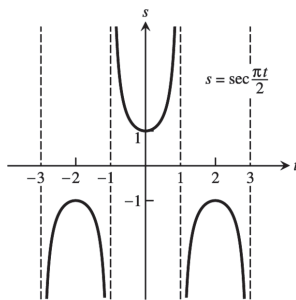
23. period =  $\frac{\pi}{2}$ , symmetric about the origin



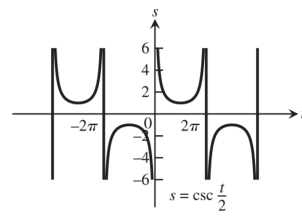
24. period = 1, symmetric about the origin



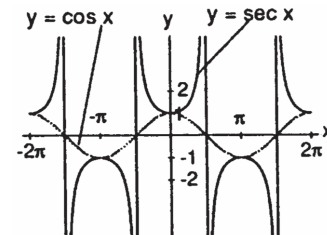
25. period = 4, symmetric about the  $s$ -axis



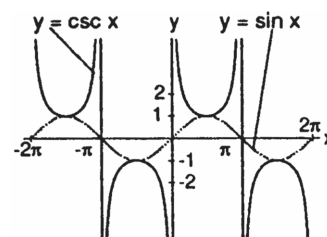
26. period =  $4\pi$ , symmetric about the origin



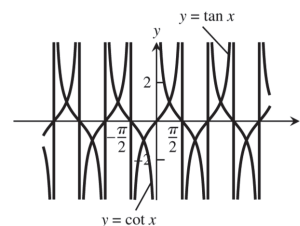
27. (a)  $\cos x$  and  $\sec x$  are positive for  $x$  in the interval  $(-\frac{\pi}{2}, \frac{\pi}{2})$ ; and  $\cos x$  and  $\sec x$  are negative for  $x$  in the intervals  $(-\frac{3\pi}{2}, -\frac{\pi}{2})$  and  $(\frac{\pi}{2}, \frac{3\pi}{2})$ .  $\sec x$  is undefined when  $\cos x$  is 0. The range of  $\sec x$  is  $(-\infty, -1] \cup [1, \infty)$ ; the range of  $\cos x$  is  $[-1, 1]$ .



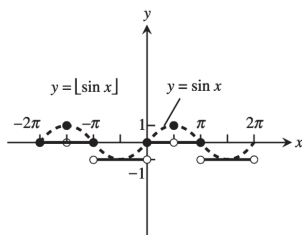
- (b)  $\sin x$  and  $\csc x$  are positive for  $x$  in the intervals  $(-\frac{3\pi}{2}, -\pi)$  and  $(0, \pi)$ ; and  $\sin x$  and  $\csc x$  are negative for  $x$  in the intervals  $(-\pi, 0)$  and  $(\pi, \frac{3\pi}{2})$ .  $\csc x$  is undefined when  $\sin x$  is 0. The range of  $\csc x$  is  $(-\infty, -1] \cup [1, \infty)$ ; the range of  $\sin x$  is  $[-1, 1]$ .



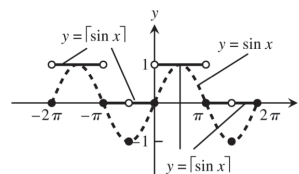
28. Since  $\cot x = \frac{1}{\tan x}$ ,  $\cot x$  is undefined when  $\tan x = 0$  and is zero when  $\tan x$  is undefined. As  $\tan x$  approaches zero through positive values,  $\cot x$  approaches infinity. Also,  $\cot x$  approaches negative infinity as  $\tan x$  approaches zero through negative values.



29.  $D: -\infty < x < \infty; R: y = -1, 0, 1$



30.  $D: -\infty < x < \infty; R: y = -1, 0, 1$



31.  $\cos\left(x - \frac{\pi}{2}\right) = \cos x \cos\left(-\frac{\pi}{2}\right) - \sin x \sin\left(-\frac{\pi}{2}\right) = (\cos x)(0) - (\sin x)(-1) = \sin x$
32.  $\cos\left(x + \frac{\pi}{2}\right) = \cos x \cos\left(\frac{\pi}{2}\right) - \sin x \sin\left(\frac{\pi}{2}\right) = (\cos x)(0) - (\sin x)(1) = -\sin x$
33.  $\sin\left(x + \frac{\pi}{2}\right) = \sin x \cos\left(\frac{\pi}{2}\right) + \cos x \sin\left(\frac{\pi}{2}\right) = (\sin x)(0) + (\cos x)(1) = \cos x$
34.  $\sin\left(x - \frac{\pi}{2}\right) = \sin x \cos\left(-\frac{\pi}{2}\right) + \cos x \sin\left(-\frac{\pi}{2}\right) = (\sin x)(0) + (\cos x)(-1) = -\cos x$
35.  $\cos(A - B) = \cos(A + (-B)) = \cos A \cos(-B) - \sin A \sin(-B) = \cos A \cos B - \sin A(-\sin B)$   
 $= \cos A \cos B + \sin A \sin B$
36.  $\sin(A - B) = \sin(A + (-B)) = \sin A \cos(-B) + \cos A \sin(-B) = \sin A \cos B + \cos A(-\sin B)$   
 $= \sin A \cos B - \cos A \sin B$
37. If  $B = A$ ,  $A - B = 0 \Rightarrow \cos(A - B) = \cos 0 = 1$ . Also  $\cos(A - B) = \cos(A - A) = \cos A \cos A + \sin A \sin A$   
 $= \cos^2 A + \sin^2 A$ . Therefore,  $\cos^2 A + \sin^2 A = 1$ .
38. If  $B = 2\pi$ , then  $\cos(A + 2\pi) = \cos A \cos 2\pi - \sin A \sin 2\pi = (\cos A)(1) - (\sin A)(0) = \cos A$  and  
 $\sin(A + 2\pi) = \sin A \cos 2\pi + \cos A \sin 2\pi = (\sin A)(1) + (\cos A)(0) = \sin A$ . The result agrees with the fact that the cosine and sine functions have period  $2\pi$ .
39.  $\cos(\pi + x) = \cos \pi \cos x - \sin \pi \sin x = (-1)(\cos x) - (0)(\sin x) = -\cos x$